Surface Code Communication with Quantum Multiplexing

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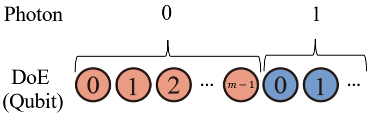
Quantum multiplexing is a method that utilize multiple degrees of freedom of photons and is known to reduce various resources in quantum information processing. We have simulated the impact of quantum multiplexing on error correction capability in optical communication using surface codes. Furthermore, we compared assignment and interleaving strategies for encoding qubits into photons. One of our strategies outperforms the performance when using random assignment.

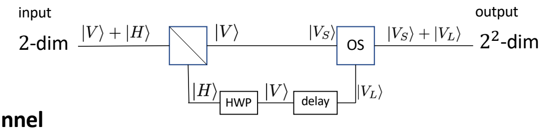
Background

- **Quantum Multiplexing**

Quantum multiplexing[1] is a method that utilizes multiple degrees of freedom of photons to reduce various resources in quantum information processing, such as the number of photons, qubits, and gates[2]



e.g. Timebin encoding



- **Erasure channel**

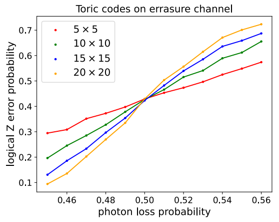
Erasure(photon loss) is the dominant source of errors in optical systems.

$$\rho \rightarrow (1-\varepsilon)\rho + \varepsilon|e\rangle\langle e|$$

where $|e\rangle \notin \mathcal{H}_2$

Correction procedure

0. Detect erasure errors
1. Replace the erased qubit with a mixed-state $$\frac{\mathbb{I}}{2} = \frac{1}{4}(\rho + X\rho X + Y\rho Y + Z\rho Z)$$
2. Apply stabilizer measurement
3. Decoding (calculate syndromes and correct)



Surface code communication

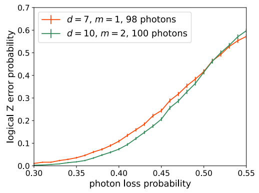
Applying QM (with m qubits in one photon) to surface code communication enables the following:

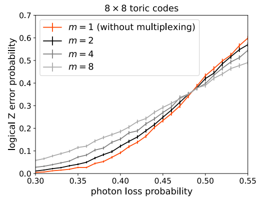
1. Sending m different codewords with the same number of photons
2. Sending $\sqrt{m}\times\sqrt{m}$ bigger codewords with the same number of photons
3. Sending original code words with $\frac{1}{m}$ photons

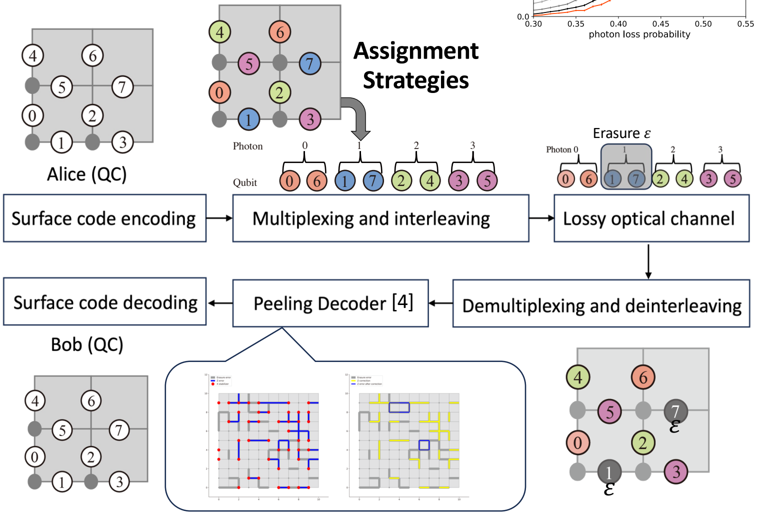
In the third case, errors are correlated between qubits encoded on the same photon

→ **Increase the logical error rate!**

We use interleaving [3] strategies to address this issue.







Assignment Strategies

Alice (QC)

Surface code encoding → Multiplexing and interleaving → Lossy optical channel

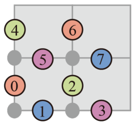
Demultiplexing and deinterleaving → Peeling Decoder [4] → Surface code decoding

Bob (QC)

Interleaving Strategies

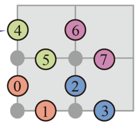
- **Strategy 1 and 2**

Maximizing/minimizing the distance between qubits in the same photon



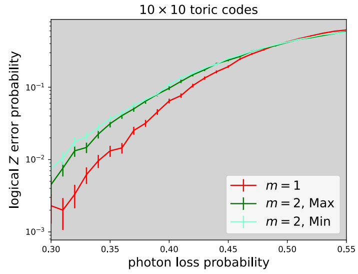
Strategy 1

(Deterministic)
Maximum distance



Strategy 2

(Deterministic)
Minimum distance



→Maximizing distance inhibits error growth

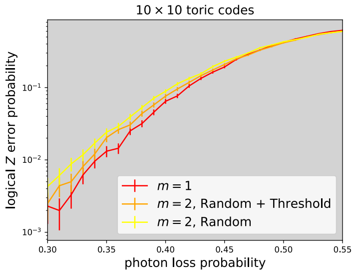
- **Strategy 3: random**

Erasure errors in multiplexed communication channels can be regarded as burst errors of length m. When dealing with burst errors in (classical), a technique called interleaving is often used. This distribute burst errors throughout the system, sometimes using **random permutations**. This can also be used for quantum communication.

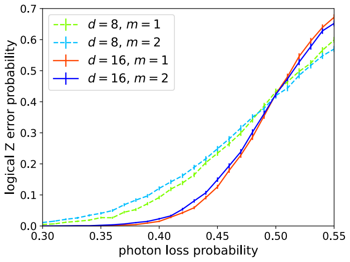
- **Strategy 4: Random + TH**

Randomness + improved distance

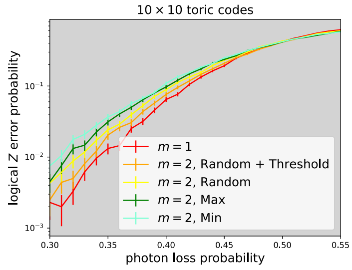
```
For photons:
    Initialize threshold
        $T = \frac{d}{2} - 1$

    Pick 1st qubit randomly
    While # of qubits in the photon < $m$
        Pick a candidate qubit $c$ randomly.
        if $c$ has a distance $d > T$
        from the nearest member,
            add $c$ to the photon
        if there is no candidate,
            update $T$ as $T - 1$
```



Result and conclusion



As the code size is increased, the performance loss due to noise correlation becomes smaller and relatively negligible.



Random + threshold outperforms other strategies

Quantum multiplexing reduces the number of photons and introduces the classical correlation in the errors affecting different qubits. This classical correlation causes an increase in the logical error rate, which can be mitigated by increasing the code size and by interleaving strategies.

Future work

- Analyzing the impact of correlations and strategies on thresholds
- Comparison with codes that are better at correcting burst errors (e.g. Quantum Reed-Solomon codes)

Reference

[1] N. Lo Piparo, W. J. Munro, and Kae Nemoto, Quantum multiplexing, Physical Review A99,022337(2019).

[2] S. Nishio, N. Lo Piparo, M. Hanks, W. J. Munro & Kae Nemoto, Resource reduction in multiplexed high-dimensional quantum Reed-Solomon codes. *Physical Review A*, 107(3), 032620 (2023).

[3] J. G. Proakis, & M. Salehi, Digital communications, vol. 1221 (1987).

[4] N. Delfosse & G. Zémor, Linear-time maximum likelihood decoding of surface codes over the quantum erasure channel. *Physical Review Research*, *2*(3), 033042 (2020).

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